URUMU DHANALAKSHMI COLLEGE
PG & RESEARCH DEPARTMENT OF MATHEMATICS
MODEL EXAMINATION- MAY 2020

SUBJECT: ALLIED MATHEMATICS-II

CLASS: I B.Sc PHYSICS

Time: 3 Hours **Max.Marks:75**

Part-A [10X2=20]
Answer all questions

1. Find approximate value of $\sqrt{98}$ using binomial expansion

2. Prove that $\frac{e^2-1}{e^2+1}=\frac{\frac{1}{1!}+\frac{1}{3!}+\frac{1}{5!}+\ldots}{1+\frac{1}{2!}+\frac{1}{4!}+\ldots.}$

3. Verify that $\frac{1}{3}\begin{bmatrix}-1 & 2 & -2\\ -2 & 1 & 2\\ 2 & 2 & 1\end{bmatrix}$ is orthogonal

4.Define hermitian matrix

5. Find the equation of the sphere whose centre is (2,-3,4) and radius 5 units

6. Find the equation of tangent plane to the sphere $x^2+y^2+z^2+6x-2y-4z=35$ at (3,4,4)

7. If $\frac{\sin\theta}{\theta}=\frac{2165}{2166}$ Show that θ is nearly equal to 3^0 1’

8. Write the series of $\text{Cos}\,\theta$ in terms of powers of θ

9. Prove that sin(ix)= isin hx

10.Prove $\cosh^2x-\sinh^2x=1$

Part-B [5x5=25]

Answer ALL questions

11a) Find the sum to infinity of the series $\frac{1}{6}+\frac{1.4}{6.12}+\frac{1.4.7}{6.12.18}+......$

OR

b) Sum to infinity of the series $1+\frac{1+3}{2!}+\frac{1+3+3^2}{3!}+\frac{1+3+3^2+3^3}{4!}+...$

12a) State and prove Cayley Hamilton theorem for A= $\begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$ and hence find A^{-1}

OR

b) Show that the matrix A= $\begin{pmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{pmatrix}$ satisfies its own characteristic equation and hence find A^4

13a) Show that the lines $\frac{x-4}{2}=\frac{y-5}{3}=\frac{z-6}{4}$ and $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ are coplanar and find the equation of the plane containing them.

OR

b) Show that the spheres $x^2+y^2+z^2=25$ and $x^2+y^2+z^2-24x-40y-18z+225=0$ touch externally and find their point of contact.

14a) Prove that $\cos 6\theta = 32\cos^6\theta - 48\cos^4\theta + 18\cos^2\theta - 1$

OR

b) Express $\frac{\sin 7\theta}{\sin\theta}$ in series of powers on $\sin\theta$

15a) Prove that $\sinh^{-1} x = \log\left[x+\sqrt{x^2+1}\right]$

OR

b) Separate into real and imaginary parts of $\tan^{-1}(\alpha + i\beta)$

Part-C [3x10=30]

Answer any THREE questions

16. Show that $\frac{1}{1.3}\frac{1}{2^2}+\frac{1}{2.4}\frac{1}{2^2}+\frac{1}{3.5}\frac{1}{2^3}+.....=\frac{5}{4}-\frac{3}{2}\log 2$

17. Find the eigen value and eigen vectors of the matrix $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$

18. Find the length and equation of Shortest distance between the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$

19, Prove that $\text{Cos}^4\theta\sin^3\theta=\frac{-1}{2^6}(\sin 7\theta+\sin 5\theta-3\sin 3\theta-3\sin\theta)$

20. If tan(A+iB) = x+iy prove that $x^2+y^2+2x\cot 2A=1$ and $x^2+y^2-2y\coth 2B+1=0$

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PG & RESEARCH DEPARTMENT OF MATHEMATICS
QUESTION BANK

SUBJECT:Allied Mathematics –II

CLASS: I B.Sc PHYSICS

UNIT-I

Part-A

1.Write the expansion of $(1-x)^{-p/q}$

2.Find approximate value of $\sqrt{98}$ using binomial expansion

3.If x is large prove that $\sqrt{x^2+4}-\sqrt{x^2+1}=\frac{3}{2x}$ nearly

4.Prove that $\frac{e^2-1}{e^2+1}=\frac{\frac{1}{1!}+\frac{1}{3!}+\frac{1}{5!}+\dots}{1+\frac{1}{2!}+\frac{1}{4!}+\dots.}$

5.Show that $\log 2-\frac{1}{2!}(\log 2)^2+\frac{1}{3!}(\log 2)^3+\dots\dots=\frac{1}{2}$

6.Prove that $\frac{e-1}{e+1}=\frac{\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+\dots}{\frac{1}{1!}+\frac{1}{3!}+\frac{1}{5!}+\dots.}$

7.Show that $\log_2 e-\log_4 e+\log_8 e-\log_{16} e+\dots.=1$

8. If x is positive, Show that $\log x=\frac{x-1}{x+1}+\frac{1}{2}\frac{x^2-1}{(x+1)^2}+\frac{1}{3}\frac{x^3-1}{(x+1)^3}+\dots.$

Part-B

1.Find the sum to infinity of the series $\frac{1}{6}+\frac{1.4}{6.12}+\frac{1.4.7}{6.12.18}+\dots\dots$

2. Find the sum to infinity of the series $1-\frac{1}{5}+\frac{1.4}{5.10}+\frac{1.4.7}{5.10.15}+\dots\dots$

3.Find the sum to infinity of the series $\frac{2.3}{3!}+\frac{3.5}{4!}+\frac{4.7}{5!}+\frac{5.9}{6!}+\ldots$

4.Sum to infinity of the series $1+\frac{1+2}{2!}+\frac{1+2+3}{3!}+\ldots$

5.Show that $\frac{5}{1.2.3}+\frac{7}{3.4.5}+\frac{9}{5.6.7}+\ldots.$ = 3 log2-1

6.Show that $\frac{1}{1.2.3}-\frac{1}{2.3.4}+\frac{1}{3.4.5}-\frac{1}{4.5.6}.+\ldots.=2\log 2-\frac{5}{4}$

Part-C

1. Find the sum to infinity of the series $\frac{7}{72}+\frac{7.28}{72.96}+\frac{7.28.49}{72.96.120}+\ldots\ldots$

2. Find the sum to infinity of the series $\frac{5}{3.6}\left(\frac{1}{4^2}\right)+\frac{5.8}{3.6.9}\left(\frac{1}{4^3}\right)+\frac{5.8.11}{3.6.9.12}\left(\frac{1}{4^4}\right)+\ldots\ldots$

3.Prove that $\sum_{0}^{\infty}\frac{5n+1}{(2n+1)!}=\frac{e}{2}+\frac{2}{e}$

4.Sum to infinity of the series $1+\frac{1+3}{2!}+\frac{1+3+3^2}{3!}+\frac{1+3+3^2+3^3}{4!}+\ldots$

5.Sum to infinity of the series $\frac{x}{1.2}-\frac{x^2}{2.3}+\frac{x^3}{3.4}+\ldots.$

6.Show that $\frac{1}{1.3}\frac{1}{2^2}+\frac{1}{2.4}\frac{1}{2^2}+\frac{1}{3.5}\frac{1}{2^3}+\ldots..=\frac{5}{4}-\frac{3}{2}\log 2$

UNIT-II

Part-A

1.Define singular matrix

2.Check whether the matrix $\begin{pmatrix}1&2&3\\2&3&1\\3&1&2\end{pmatrix}$ is non-singular?

3.Define symmetric matrix

4.Define Skew symmetric matrix

5.Check whether the matrix is $\begin{pmatrix} 1 & 1 & -1 \\ 1 & 2 & 0 \\ -1 & 0 & 5 \end{pmatrix}$ symmetric?

6.Verify that $\frac{1}{3}\begin{bmatrix} -1 & 2 & -2 \\ -2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ is orthogonal

7.Define hermitian matrix

8.Define Skew Hermitian matrix

9.Define Unitary matrix

10.Write the characteristic equation of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

11.Show that the matrix $\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$ is orthogonal

12.show that the matrix $\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \\ \frac{-i}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$ is unitary

13.State Cayley Hamilton Theorem

Part-B & Part-C

1.Find the eigen value and eigen vectors of the matrix $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$

2. Find the eigen value and eigen vectors of the matrix $\begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$

3. Find the eigen value and eigen vectors of the matrix $\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$

4.State and prove Cayley Hamilton theorem for A= $\begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$ and hence find A^{-1}

5.Show that the matrix A= $\begin{pmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{pmatrix}$ satisfies its own characteristic equation and hence find A^4

UNIT-III

Part-A

1.Show that the lines $\frac{x-4}{1}=\frac{y+3}{-4}=\frac{z+1}{7}$ and $\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$ are coplanar

2.Find the equation of the sphere whose centre is (2,-3,4) and radius 5 units

3.Find the centre and radius of sphere $3(x^2+y^2+z^2)+12x-8y-10z+10=0$

4.Find the equation of tangent plane to the sphere $x^2+y^2+z^2+6x-2y-4z=35$ at (3,4,4)

5.Define skew lines

6.What do you mean by coplanar lines

Part-B

1.Prove that the lines $\frac{x-4}{1}=\frac{y+3}{-4}=\frac{z+1}{7}$ and $\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$ intersect and find the coordinates of their point of intersection. Find also the equation of the plane containing them

2.Show that the lines $\frac{x-4}{2}=\frac{y-5}{3}=\frac{z-6}{4}$ and $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ are coplanar and find the equation of the plane containing them.

3.Show that the plane 2x-2y+z=9 touches the sphere $x^2+y^2+z^2+2x+2y-7=0$ and find the point of contact

4.Show that the spheres $x^2+y^2+z^2=25$ and $x^2+y^2+z^2-24x-40y-18z+225=0$ touch externally and find their point of contact.

Part-C

1.Find the length and equation of Shortest distance between the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$

2.Find the equation of the sphere for which the circle $2x+3y+4z=8, x^2+y^2+z^2+7y-2z+2=0$ is a great circle.

3.Find the centre and radius of the circle $x^2+y^2+z^2=9, x+y+z=3$

UNIT-IV

Part-A

1.Write the formula to expand $\sin n\theta$ in series of powers of $\sin\theta$ and $\cos\theta$

2.Write the series of $\sin\theta$ in terms of powers of θ

3. Write the formula to expand $\tan n\theta$ in series of powers of $\tan n\theta$

4.If $\frac{\sin\theta}{\theta}=\frac{2165}{2166}$ Show that θ is nearly equal to 3^0 1'

5.Evaluate $\lim_{x\to 0}\frac{\sin x-\tan x}{x^3}$

Part-B

1.Express $\frac{\sin 7\theta}{\sin\theta}$ in series of powers on $\sin\theta$

2.Prove that $\cos 6\theta=32\cos^6\theta-48\cos^4\theta+18\cos^2\theta-1$

3.Prove $\text{Cos}7\theta\sec\theta=64\cos^6\theta-112\cos^4\theta+56\cos^2\theta-7$

Part-C

1.Prove that $\sin^7\theta=\frac{1}{4}(35\sin\theta-21\sin 3\theta+7\sin 5\theta-\sin 7\theta)$

2.Expand $\cos^8\theta$ in a series of cosines of multiples of θ

3.Prove that $\text{Cos}^4\theta\sin^3\theta=\frac{-1}{2^6}(\sin 7\theta+\sin 5\theta-3\sin 3\theta-3\sin\theta)$

4.Expand $\text{Sin}^4\theta\cos^2\theta$ in series of cosines of multiples of θ

UNIT-V

Part-A

1.Prove that sin(ix)= isin hx

2.Prove $\cosh^2 x - \sinh^2 x = 1$

3.separate into real and imaginary parts of sin(x+iy)

4.Separate into real and imaginary parts of cosh(x+iy)

5.Express coshx in terms of e^x & e^{-x}

Part-B & Part-C

1.Prove that $\sinh^{-1} x = \log\left[x + \sqrt{x^2+1}\right]$

2. Prove that $\cosh^{-1} x = \log\left[x + \sqrt{x^2-1}\right]$

3.Prove that $\tanh^{-1} x = \frac{1}{2}\log\frac{1+x}{1-x}$

4.separate into real and imaginary parts of a) tan(x+iy) b) tanh(x+iy)

5.Separate into real and imaginary parts of $\tan^{-1}(\alpha + i\beta)$

6.If sin(A+iB)=x+iy then prove that

(i) $\frac{x^2}{\cosh^2 B} + \frac{y^2}{\sinh^2 B} = 1$ and (ii) $\frac{x^2}{\sin^2 A} - \frac{y^2}{\cos^2 A} = 1$

7.If tan(A+iB) = x+iy prove that

$x^2+y^2+2x\cot 2A = 1$ and $x^2+y^2-2y\coth 2B+1=0$

8.If x+iy = c Cos(A-iB) show that

(i) $\frac{x^2}{c^2\cosh^2 B} + \frac{y^2}{c^2\sinh^2 B} = 1$ and (ii) $\frac{x^2}{c^2\cos^2 A} - \frac{y^2}{c^2\sin^2 A} = 1$