URUMU DHANALAKSHMI COLLEGE

PG & RESEARCH DEPARTMENT OF MATHEMATICS

QUESTION BANK

SUBJECT: ABSTRACT ALGEBRA **CLASS: III B.Sc MATHEMATICS**

UNIT-1

Part-A

1.Define group

2.Define idempotent element

3.Define permutation

4.Define symmetric group

5.Define disjoint cycle

6.Define Transposition

7.Define alternating group

8.Give examples of Even and Odd permutations

9.Define abelian group

Part-B

1.Let G be a group. Let a,b ϵ G. Then $(ab)^{-1}=b^{-1}a^{-1}$ and $(a^{-1})^{-1}$

2.Provethat (Zn,+n) is a group

3.Let G be a group in which $(ab)^m=a^mb^m$ for three consecutive integers and for all a,b ϵ G. Then G is abelian.

4Let(H, .) and (K,*) be groups. Define a binary operation # on HxK by (h1,k1)#(h2,k2)=(h1h2,k1*k2) then Prove that HxK is a group

5.Let G be a finite set with an associative binary operation defined on G in which both cancellation laws hold good. Then G is a group.

6.Any permuatation can be expressed as product of disjoint cycles.

7.Prove that (i)product of two even permutations is an even permutation.(ii)product of two odd permutations is an even permutation.(iii)Product of even and odd permutations is odd permutation.

8.Prove that (i)Inverse of an even permutation is even (ii) Inverse of an Odd permutation is odd (iii)The identity permutation is an even permutation

Part-C

1.If a permutation p ϵ Sn is a product of r transpositions and also a product of s transpositionsthen either r and s are both even or both odd.

2.Let An be the set of all even permutations in Sn. Then An is a group containing n!/2 permutations.

UNIT-II

Part-A

1.Define Subgroup

2.What are Improper subgroup

3.Define Centre of a group

4.Define Normaliser of a group

5.Define Cyclic subgroup

6.define Cyclic group

7.define Left coset

8.define Index

9.State Lagrange's theorem

10.State Euler's theorem

11.State Fermat's theorem

Part-B

1.A non-empty subset H of a group G is a subgroup of G iff a,b ϵ H => ab^{-1} ϵH

2. If H and K are subgroups of group G then H∩K is also a subgroup of G

3.The Union of two subgroups of a group G is a subgroup iff one is contained in other

4.Let G be a group. Let $H = \{a / a \in G \text{ and } ax = xa \text{ for all } x \in G\}$. Show that H is a subgroup of G

5.let G be a group and a be a fixed element of G. Let Ha = $\{x / x \in G \text{ and } ax = xa\}$. Show that Ha is a subgroup of G.

6.Define Cyclic Group. Prove that any cyclic group is abelian

7.Let G be a group and a∈G. then the order of a is the same as the order of the cyclic group generated by 'a'

8.Let G be a group and a be an element of order n in G. then a^m=e iff n divides m.

9. Prove that a subgroup of a cyclic group is cyclic.

10.Let H be a subgroup of G. Then (i)anytwo left cosets of H are either identical or disjoint. (ii) union of all left cosets of H is G (iii) the number of elements in any left coset aH is the same as the number of elements in H.

11.Let H be a subgroup of G. The prove that the number of left cosets of H is the same as the number of right cosets of H.

12.State and prove Lagrange's theorem

13.State and prove Euler's theorem

14.A group G has no proper subgroups if it is a cyclic group of prime order.

15.Let H and K be two subgroups of G. Then prove that $|HK| = \frac{|H||K|}{|H \cap K|}$

Part-C

1.Let A and B be two subgroups of a group G. then AB is a subgroup of G iff AB = BA

2.Let G be a group and a,b∈G. Then (i) order of a = order of a^{-1}

(ii)order of a =order of $b^{-1}ab$ (iii) order of ab = order of ba

3.Let G be a group and H be a subgroup of G. Then (i) a ϵ G => aH=H (ii) aH = bH => $a^{-1}b$ϵ H

(iii) aϵ bH => a^{-1}ϵHb^{-1} (iv) aϵ bH => aH =bH

UNIT-III

Part-A

1.Define normal subgroup

2.Define quotient group

3.Define isomorphism of groups

4.Define Automorphism

5.Define Inner Automorphism

6.Define Homomorphism

7.Define epimorphism, monomorphism, endomorphism.

8.Define kernel of a homomorphism

9. Let f:G→G' be a homomorphism. Then prove that f is 1-1 iff ker f={e}

10.Prove: Any homomorphic image of a cyclic group is cyclic

Part-B

1.Prove that every subgroup of an abelian group is a normal subgroup.

2.Prove: A subgroup of N of G is normal iff the product of two right cosets of N is again a right coset of N

3.Let N be a normal subgroup of a group. Then prove that G/N is a group under the operationNaNb=Nab

4.Prove that Isomorphism is an equivalence relation among groups

5.Let f:G→G'be an isomorphism. Let a∈G. Then the order of a is equal to to the order of f(a) [OR]

Prove that Isomorphism preserves the order of each element of a group.

6.Let f:G→G' be an isomorphism. If G is cyclic then Prove that G' is also cyclic

7.Any infinite cyclic group G is isomorphic to (Z,+).Any finite cyclic group of order n is isomorphic to (Zn,+n)

8.Prove: The number of automorphisms of a cyclic group of order n is $\varphi(n)$

9.Let f:G→G' be a homomorphism. Then prove that the kernel K of f is a normal subgroup of G.

Part-C

1. State and prove Cyley's theorem. [OR] Any finite group is isomorphic to a group of permutations.

2.For any group, prove that

(i) Aut G is a group under composition of functions. {ii) I(G) is a normal subgroup of Aut G

3.Let f:G→G' be a homomorphism. Then prove that

(i)If His a subgroup of G then f(H) is a subgroup of G' (ii) If H is normal in G then f(H) is normal in f(H)

(iii)If H'is a subgroup of G' then f^{-1}(H') is a subgroup of G'(iv) If H' is normal in G' then f^{-1}(H) is normal in G

4.State and prove fundamental theorem of homomorphism for groups [OR]

Let f:G→G' be an epimorphism. Let K be the kernel of f. then Prove that G/K $\cong G'$

UNIT-IV

Part-A

1.Define ring of Gaussian integers.

2.Define Boolean Ring

3.Define Isomorphism of rings

4.Define Unit

5.Define CommutativeRing

5.Define Skew field (Division Ring)

6.In a ring with identity, prove that Identity element is unique.

7.Define Field

8.What is a zero divisor?

9. Prove that any unit in R cannot be a zero divisor.

10.Define Integral Domain

11.Define subring

12.Define Subfield

13.Define Ideal

14.Define principal ideal

15.Define Principal ideal domain

Part-B

1.If R is a ring such that $a^2=a$ for all $a\epsilon R$, prove that (i)a+a=0 (ii) a+b=0 => a=b (iii)ab=ba

2.Let R be a ring with identity. The set of all units in R is a group under multiplication

3.Prove : A ring R has no zero divisors iff cancellation law is valid in R

4.prove that Zn is an integral domain iff n is prime

5.Prove: Any field F is an integral domain

6.Prove that A non-empty subset S of a ring R us a subring iff a-b ϵ S => a-b ϵ S and ab ϵ S

7.Prove that the intersection of two subrings of a Ring R is a subring of R.

8.Prove that A field has ho proper ideals[OR] The only ideals of Field F are {0} and F

9.Prove: A Commutative ring R is a field iff R has no proper ideals.

Part-C

1.Prove that any finite integral domain is a field

2.Prove that a finite commutative ring R withour zero divisors is a field.

3.The characteristic of an integral domain D is either 0 or a prime number

4..Let R be a ring and I be a subgroup of (R,+). The multiplication in R/I given by (I+a)(I+b)=I+ab is well defined iff I is any ideal of R.

UNIT-V

Part-A

1.Define Maximal ideal

2.Define prime ideal

3.Define homomorphism, monomorphism,epimorphism and endomorphism w.r.to rings

4.Define kernel of a homomorphism of rings

5.Define associates

6.Define U.F.D

7.Define Euclidean Domain

Part-B

1. let R be a commutative ring with identity. Let P be an ideal of R. Then P is a prime ideal $\Leftrightarrow$ R/P is an integral domain.

2.Let R be an integral domain. Let a and b be two non zero elements of r. Then a and b are associates iff a=bu where u is a unit in R.

3.Prove that Every ideal of an Euclidean domain is a principal ideal.

4.Prove: Any Euclidean domain R has an identity elelment.

5.Prove: Any Euclidean domain R is a principal ideal domain.

6.let R be an Euclidean domain. Let a and b be two non-zero elements of R. Then Prove that

(i) b is not a unit in R $\Rightarrow$ d(a)<d(ab) and (ii) b is a unit in R $\Rightarrow$ d(a) =d(ab)

7.Let a be a non-zero element of an Euclidean domain R. Then a is a unit in R iff d(a)=d(b)

Part-C

1.let R be a commutative ring with identity. An ideal M of R is maximal iff R/M is a field

2.State and prove fundamental theorem of homomorphism [OR] Let R and R' be rings. Let f:R→R' be an epimorphism. Let K be the kernel of f. Then prove that R/K $\approx$ R'

3.Any Euclidean domain R is a UFD

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Assignment-1

SUBJECT:GRAPH THEORY **CLASS: III B.Sc MATHEMATICS**

Answer all questions

Part-A

1.State Konigsberg bridge problem

2.Define Pseudograph. Give an example.

3.Define Bipartite graph

4.Define spanning subgraph. Give an example

5.Define induced subgraph

Part-B

6.In any graph G show that the number of points of odd degree is even.

7.Show that in any group of two or more people, there are always two with exactly same number of friends inside the group.

8.Prove that $\delta \leq \frac{2q}{p} \leq \Delta$

9.Prove that any self complementary graph has 4n or 4n+1 points.

Part-C

10.The maximum number of lines among all p point graphs with no triangles is $\left[\frac{p^2}{4}\right]$

11(a)Prove that $\Gamma(G) = \Gamma(\overline{G})$

(b)With usual notations, Prove that $\alpha + \beta = p$.

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Assignment-2

SUBJECT:GRAPH THEORY **CLASS: III B.Sc MATHEMATICS**

Answer all questions

Part-A

1.Define Adjacency matrix. Give example

3.Define walk

3.Define connected graph

4.Define cut point

5.Define Euler graph

Part-B

6.Prove:In a graph G, any u-v walk contains a u-v path.

7.P.T a closed walk of odd length contains a cycle.

8.Prove that a graph G with atleast two points is bipartite iff all its cycles are of even length.

9.Prove:If G is a graph in which the degree of every vertex is atleast two then G contains a cycle.

Part-C

10. Prove:A graph G is connected iff for any partition of V into subsets V_1 and V_2 there is a line of G joining a point of V_1 to a point of V_2

11.Prove that the following statements are equivalent for a connected graph

(i)G is Eulerian

(ii)Every point of G has even degree

(iii)The set of edges of G can be partitioned into cycles

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Assignment-3

SUBJECT:GRAPH THEORY **CLASS: III B.Sc MATHEMATICS**

Answer all questions

Part-A

1.Define Hamiltonian graph

2.Define tree

3.Define centre of a tree

4.Prove that every Hamiltonian graph is 2-connected

5.Every connected graph has a spanning tree

Part-B

6. Prove that every connected graph has a spanning tree.

7.Prove that C(G) is well defined.

8.Prove: A graph is Hamiltonian iff its closure is Hamiltonian.

9.Every tree has a centre consisting of either one point or two adjacent points.

Part-C

10.If G is a graph $p \geq 3$ vertices and $\delta \geq p/2$ then prove that G is Hamiltonian.

11.Let G be a (p,q) graph. Prove that the following statements are equivalent.

(i) G is a tree (ii)Every two points of G are joined by a unique path

(iii)G is connected and p=q+1 (iv).G is acyclic and p=q+1

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Assignment-4

SUBJECT:GRAPH THEORY **CLASS: III B.Sc MATHEMATICS**

Answer all questions

Part-A

1.\Define embedded graph

2.Define Planar graph

3.Define plane graph

4.Define triangulated graph

5.Define homeomorphic graph

Part-B

6.Prove that K_5 is non-planar

7.Prove that a graph can be embedded in a sphere iff it can be embedded in a plane.

8.Prove:Every 2-connected plane graph can be embedded in the plane so that any specified face is the exterior face.

9.Every planar graph G with atleast 3 points is a subgraph of a triangulated graph with same number of points

Part-C

10.P.T every polyhedran has atleast two faces with the same number of edges on the boundary.

11.If G is a connected plane graph having V,E and F as the set of vertices, edges and faces respectively, then prove that $|V|-|E|+|F|=2$

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Assignment-5

SUBJECT:GRAPH THEORY **CLASS: III B.Sc MATHEMATICS**

Answer all questions

Part-A

1.Define digraph

2.Define Indegree

3.Define Outdegree

4.Define isomorphism of digraph

5.Define weighted graph

Part-B

6.If two digraphs are isomorphic then prove that corresponding points have the same degree pair

7.Prove: In a digraph D, sum of the indegrees of all the vertices is equal to the sum of their out degrees, each sum being equal to the number of arcs in D

Part-C

8.Explain Kruskal's Algorithm

9.Explain Dijkstra's algorithm

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QUESTION BANK

SUBJECT:GRAPH THEORY **CLASS: III B.Sc MATHEMATICS**

UNIT-1

Part-A

1.State Konigsberg bridge problem

2.Define Graph

3.Draw Peterson graph

4.Defiine loop

5.Define Pseudograph

6.Define Bipartite graph

7.Define degree of a vertex

8.Define subgraph

9.Define spanning subgraph

10.Define induced subgraph

11.Define complement of a graph

12.Define automorphism graph

13.Define Independent set

14.Define Line covering

Part-B

1.Show that maximum number of lines in a graph with n vertices is n(n-1)/2.

2.In any graph G show that the number of points of odd degree is even.

3.Show that in any group of two or more people, there are always two with exactly same number of friends inside the group.

4.Prove that $\delta \leq \frac{2q}{p} \leq \Delta$

5.Let G be a k-regular graph with bipartition (V_1,V_2) and k>0. Prove that $|V_1| = |V_2|$

6.Prove that any self complementary graph has 4n or 4n+1 points.

7.Prove that $\Gamma(G) = \Gamma(\overline{G})$

8.With usual notations, Prove that $\alpha + \beta = p$.

Part-C

1.The maximum number of lines among all p point graphs with no triangles is $\left[\frac{p^2}{4}\right]$

UNIT-II

Part-A

1.Define Adjacency matrix

2.Define Incidence matrix

3.Define walk

4.Define trail

5.Define path

6.Define connected graph

7.Define cut point

8.Define bridge

9.Define Euler graph

Part-B

1.Let G_1 be a (p_1,q_1) graph and G_2 be (p_2,q_2) graph. Prove that

(i)$G_1 x G_2$ is a (p_1p_2,$q_1p_2+q_2p_1$) graph & (ii)$G_1[G_2]$ is a (p_1p_2, $p_1q_2+p_2^2q_1$) graph.

2.Prove:In a graph G, any u-v walk contains a u-v path.

3.P.T a closed walk of odd length contains a cycle.

4.P.T a graph G with p points and $\delta \geq \frac{p-1}{2}$ is connected

5.Prove that a graph G with atleast two points is bipartite iff all its cycles are of even length.

6.Let v be a point of a connected graph G. Then prove that following statements are equivalent.

(i) v is a cut-point of G

(ii)There exists a partition of V-{v} into subsets U and W such that for each u∈U and w∈W, the point v is on every u-w path

(iii)There exists two points u and w distinct from v such that v is on every u-w path.

7.Prove:A line x of a connected graph G is a bridge iff x is not on any cycle of G

8.Prove: Every non-trivial connected graphs has atleast two points which are not cutpoints.

9.Prove:If G is a graph in which the degree of every vertex is atleast two then G contains a cycle.

10.Prove that an Eulerian graph G is arbitrary traversable from a vertex v in G iff every cycle in G contains v (Fleury's Algorithm)

Part-C

1. Prove:A graph G is connected iff for any partition of V into subsets V_1 and V_2 there is a line of G joining a point of V_1 to a point of V_2

2.Prove that the following statements are equivalent for a connected graph

(i)G is Eulerian

(ii)Every point of G has even degree

(iii)The set of edges of G can be partitioned into cycles

UNIT-III

Part-A

1.Define Hamiltonian graph

2.Define tree

3.Define centre of a tree

4.Prove that every Hamiltonian graph is 2-connected

5.Every connected graph has a spanning tree

Part-B

1.Prove that C(G) is well defined.

2.Prove: A graph is Hamiltonian iff its closure is Hamiltonian.

3.Prove that every connected graph has a spanning tree.

4.Let T be a spanning tree of a connected graph G. Let x=uv be an edge of G not in T. Then prove that T+x contains a unique cycle.

5.Every tree has a centre consisting of either one point or two adjacent points.

Part-C

1.If G is a graph p$\geq$ 3 vertices and $\delta \geq p/2$ then prove that G is Hamiltonian.

2.Let G be a (p,q) graph. Prove that the following statements are equivalent.

(i) G is a tree (ii)Every two points of G are joined by a unique path

3.G is connected and p=q+1 4.G is acyclic and p=q+1

UNIT-IV

Part-A

1.\Define embedded graph

2.Define Planar graph

3.Define plane graph

4.Define triangulated graph

5.Define homeomorphic graph

Part-B

1.Prove that K_5 is non-planar

2.Prove that a graph can be embedded in a sphere iff it can be embedded in a plane.

3.Prove:Every 2-connected plane graph can be embedded in the plane so that any specified face is the exterior face.

4.In any connected plane (p,q) graph ($p\geq3$) with r faces $q\geq3r/2$ and $q\leq3p-6$

5.Every planar graph G with atleast 3 points is a subgraph of a triangulated graph with same number of points

6.If a (p_1,q_1) graph and (p_2,q_2) graph are homeomorphic, then prove that $p_1+q_2=p_2+q_1$

Part-C

1.P.T every polyhedran has atleast two faces with the same number of edges on the boundary.

2.State and prove Euler theorem (Or)

If G is a connected plane graph having V,E and F as the set of vertices, edges and faces respectively, then prove that $|V|-|E|+|F|=2$

UNIT-V

Part-A

1.Define digraph

2.Define Indegree

3.Define Outdegree

4.Define isomorphism of digraph

5.Define weighted graph

Part-B

1.If two digraphs are isomorphic then prove that corresponding points have the same degree pair

2.Prove: In a digraph D, sum of the indegrees of all the vertices is equal to the sum of their out degrees, each sum being equal to the number of arcs in D

Part-C

1.Explain Kruskal's Algorithm

2.Explain Dijkstra's algorithm

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Assignment - 1

SUBJECT: ABSTRACT ALGEBRA **CLASS: III B.Sc MATHEMATICS**

Part-A

1.Define idempotent elelment

2.Define symmetric group

3.Define Transposition

4.Define alternating group

5.Define abelian group

Part-B

6.Let G be a group in which $(ab)^m=a^mb^m$ for three consecutive integers and for all a,b ϵ G. Then G is abelian.

7.Let G be a finite set with an associative binary operation defined on G in which both cancellation laws hold good. Then G is a group.

8.Any permutation can be expressed as product of disjoint cycles.

9.Prove that (i)product of two even permutations is an even permutation.(ii)product of two odd permutations is an even permutation.(iii)Product of even and odd permutations is odd permutation.

Part-C

10.If a permutation p ϵ Sn is a product of r transpositions and also a product of s transpositionsthen either r and s are both even or both odd.

11.Let An be the set of all even permutations in Sn. Then An is a group containing n!/2 permutations.

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Assignment - 2

SUBJECT: ABSTRACT ALGEBRA **CLASS: III B.Sc MATHEMATICS**

Part-A

1.Define Centre of a group

2.Define Normaliser of a group

3.Define Cyclic group

4.Define Left coset

5.State Euler's theorem

Part-B

6. Prove that a subgroup of a cyclic group is cyclic.

7.Let H be a subgroup of G. The prove that the number of left cosets of H is the same as the number of right cosets of H.

8.State and prove Euler's theorem

9.Let H and K be two subgroups of G. Then prove that $|HK| = \frac{|H||K|}{|H \cap K|}$

Part-C

10.Let A and B be two subgroups of a group G. then AB is a subgroup of G iff AB = BA

11.Let G be a group and a,b∈G. Then (i) order of a = order of a^{-1}

(ii)order of a =order of $b^{-1}ab$ (iii) order of ab = order of ba

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Assignment - 3

SUBJECT: ABSTRACT ALGEBRA **CLASS: III B.Sc MATHEMATICS**

Part-A

1.Define normal subgroup

2.Define quotient group

3.Define Automorphism

4. Let f:G→G' be a homomorphism. Then prove that f is 1-1 iff ker f={e}

5.Prove: Any homomorphic image of a cyclic group is cyclic

Part-B

6.Prove that every subgroup of an abelian group is a normal subgroup.

7.Prove: A subgroup of N of G is normal iff the product of two right cosets of N is again a right coset of N

8. Prove that Isomorphism preserves the order of each element of a group.

9.Any finite cyclic group of order n is isomorphic to (Zn,+n)

Part-C

10. Prove that any finite group is isomorphic to a group of permutations.

11.Let f:G→G' be an epimorphism. Let K be the kernel of f. then Prove that G/K $\cong G'$

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Assignment - 4

SUBJECT: ABSTRACT ALGEBRA **CLASS: III B.Sc MATHEMATICS**

Part-A

1.Define Boolean Ring

2.Define Skew field (Division Ring)

3.Define Integral Domain

4.Define Ideal

5.Define Principal ideal domain

Part-B

6.prove that Zn is an integral domain iff n is prime

7.Prove: Any field F is an integral domain

8.Prove that the intersection of two subrings of a Ring R is a subring of R.

9.Prove that A field has no proper ideals

Part-C

10.Prove that any finite integral domain is a field

11.Prove:The characteristic of an integral domain D is either 0 or a prime number

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Assignment - 5

SUBJECT: ABSTRACT ALGEBRA **CLASS: III B.Sc MATHEMATICS**

Part-A

1.Define Maximal ideal

2.Define prime ideal

3.Define kernel of a homomorphism of rings

4.Define associates

5.Define U.F.D

Part-B

6. let R be a commutative ring with identity. Let P be an ideal of R. Then P is a prime ideal $\Leftrightarrow$ R/P is an integral domain.

7.Prove that Every ideal of an Euclidean domain is a principal ideal.

8.Prove: Any Euclidean domain R is a principal ideal domain.

9.let R be an Euclidean domain. Let a and b be two non-zero elements of R. Then Prove that

(i) b is not a unit in R $\Rightarrow$ d(a)<d(ab) and (ii) b is a unit in R $\Rightarrow$ d(a) =d(ab)

Part-C

10.let R be a commutative ring with identity.Prove: An ideal M of R is maximal iff R/M is a field

11Prove that Any Euclidean domain R is a UFD